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MHD NATURAL CONVECTION FLOW WITH RADIATIVE HEAT TRANSFER PAST AN IMPULSIVELY MOVING VERTICAL PLATE WITH RAMPED TEMPERATURE IN THE PRESENCE OF HALL CURRENT AND THERMAL DIFFUSION

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An investigation on an unsteady MHD natural convection flow with radiative heat transfer of a viscous, incompressible, electrically conducting and optically thick fluid past an impulsively moving vertical plate with ramped temperature in a porous medium in the presence of a Hall current and thermal diffusion is carried out. An exact solution of momentum and energy equations, under Boussinesq and Rosseland approximations, is obtained in a closed form by the Laplace transform technique for both ramped temperature and isothermal plates. Expressions for the skin friction and Nusselt number for both ramped temperature and isothermal plates are also derived. The numerical values of fluid velocity and fluid temperature are displayed graphically versus the boundary layer coordinate y for various values of pertinent flow parameters for both ramped temperature and isothermal plates. The numerical values of the skin friction due to primary and secondary flows are presented in tabular form for various values of pertinent flow parameters.

Key words: MHD natural convection, Hall current, ramped temperature, thermal diffusion, thermal radiation.

1. Introduction

Natural convection flows are frequently encountered in science and technological problems such as chemical catalytic reactors, nuclear waste repositories, petroleum reservoirs, fiber and granular insulation, geothermal systems etc. Natural convection flows from bodies with different geometries are extensively investigated as it is evident from review articles and books published so far (Ede, 1967; Gebhart, 1973; Jaluria, 1980; Raithby and Hollands, 1985). A convective heat transfer flow from bodies with different geometries embedded in a porous medium is of significant importance due to its varied and wide applications in many areas of science and technology, namely, drying of porous solids, thermal insulation, enhanced recovery of oil and gases, cooling of nuclear reactors, underground energy transport etc. Keeping in view the importance of such fluid flow problems, a number of investigations on natural convection flow near a vertical plate embedded in a porous medium have been carried out. Mention may be made of research studies of Cheng and Minkowycz (1977), Nakayama and Koyama (1987), Lai and Kulacki (1991) and Hsieh *et al.* (1993). Comprehensive reviews of free convection flows with heat and mass transfer in porous media are well presented by Pop and Ingham (2002), Vafai (2005) and Nield and Bejan (2006). An investigation of a hydromagnetic free convection flow in a porous medium under different conditions has been carried out by several researchers due to a significant effect of the magnetic field on the boundary layer control, plasma studies, geothermal energy extraction, metallurgy, petroleum and chemical engineering etc and on the performance of so many engineering devices using electrically conducting fluids, viz., MHD energy generators, MHD pumps, MHD accelerators, MHD flow-meters, nuclear reactors using liquid metal coolants

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etc. Raptis and Kafousias (1982) studied a steady free convection flow past an infinite vertical porous plate through a porous medium in the presence of a magnetic field. Raptis (1986) investigated an unsteady two-dimensional natural convection flow past an infinite vertical porous plate embedded in a porous medium in the presence of a magnetic field. Chamkha (1997a) studied a transient MHD free convection flow through a porous medium supported by a surface. Chamkha (1997b) also investigated a hydromagnetic natural convection flow from an isothermal inclined surface adjacent to a thermally stratified porous medium. Aldoss *et al*. (1995) considered a combined free and forced convection flow from a vertical plate embedded in a porous medium in the presence of a magnetic field. Kim (2000) studied an MHD natural convection flow past a moving vertical plate embedded in a porous medium. Ibrahim *et al.* (2004) studied an unsteady hydromagnetic free convection flow of a micro-polar fluid and heat transfer past a vertical porous plate through a porous medium in the presence of thermal and mass diffusion with a constant heat source. Makinde and Sibanda (2008) investigated a hydromagnetic mixed convective flow with heat and mass transfer past a vertical plate embedded in a porous medium with constant wall suction. Makinde (2009) considered a hydromagnetic mixed convection flow and mass transfer past a vertical porous plate with constant heat flux embedded in a porous medium.

In all these investigations, the effects of radiation are not taken into account. Radiative heat transfer along with free convection is important in many areas of science and engineering, viz., glass production, furnace design, electric power generation, thermo-nuclear fusion, casting and levitation, high temperature aerodynamics, propulsion systems, plasma physics, space flight, solar power technology, spacecraft re-entry aerothermodynamics, etc. In many practical applications, depending on the surface properties and configuration, radiative heat transfer is often comparable with that of convective heat transfer. It is worthy of note that unlike convection/conduction the governing equations taking into account radiative heat transfer become quite complicated and hence many difficulties arise while solving such equations. However, some reasonable approximations are proposed to solve the governing equations with radiative heat transfer. The text book by Sparrow and Cess (1970) describes the essential features of radiative heat transfer. Chang *et al*. (1983) investigated a natural convection flow with radiative heat transfer in two-dimensional complex enclosures. Cess (1966) studied laminar free convection along a vertical isothermal plate with thermal radiation using the Rosseland diffusion approximation. Hossain and Takhar (1996) considered radiation effects on a mixed convection boundary layer flow along a vertical plate with uniform surface temperature using the Rosseland flux model. Chamkha (1997c) analyzed solar radiation assisted natural convection in a uniform porous medium supported by a vertical flat plate. Chamkha *et al*. (2001) studied a laminar free convection flow of air past a semi-infinite vertical plate in the presence of chemical species concentration and thermal radiation. Muthucumaraswamy and Ganesan (2003) investigated radiation effects on flow past an impulsively started vertical plate with variable temperature. Ghosh and Bég (2008) discussed the effects of radiation on a transient free convection flow past an infinite hot vertical impulsively moving plate in a porous medium. Chamkha (2000) discussed thermal radiation and buoyancy effects on a hydromagnetic flow over an accelerating permeable surface with heat source or sink. Raptis and Massalas (1998) studied an oscillatory magnetohydrodynamic flow of a gray, absorbing-emitting fluid with a non-scattering medium past a flat plate in the presence of radiation assuming the Rosseland approximation. Azzam (2002) considered radiation effects on an MHD mixed convection flow past a semi-infinite moving vertical plate for high temperature differences. Cookey *et al*. (2003) analyzed the influence of viscous dissipations and radiation past an infinite heated vertical plate in a porous medium with time-dependent suction. Mahmoud Mostafa (2009) discussed thermal radiation effects on an unsteady MHD free convection flow past an infinite vertical porous plate taking into account the effects of viscous dissipation. Ogulu and Makinde (2009) investigated an unsteady hydromagnetic free convection flow of a dissipative and radiative fluid past a vertical plate with constant heat flux.

In all these investigations, an analytical or numerical solution is obtained assuming conditions for fluid velocity and temperature at the plate as continuous and well defined. However, there exist several practical problems which may require non-uniform or arbitrary wall conditions. Keeping this fact in view, Hayday *et al.* (1967), Kao (1975), Kelleher (1971) and Lee and Yovanovich (1991) investigated a free convection flow from a vertical plate with step discontinuities in the surface temperature. Recently, Patra *et* *al*. (2012) investigated the effects of radiation on a natural convection flow of a viscous and incompressible fluid near a vertical flat plate with ramped temperature. They compared the effects of radiative heat transfer on a natural convection flow near a ramped temperature plate with the flow near an isothermal plate. It is well known that when density of an electrically conducting fluid is low and/or the applied magnetic field is strong, effects of the Hall current become significant. The Hall current plays an important role in determining flow-features of the problem because it induces a secondary flow in the flow-field. Therefore, it is appropriate to study the effects of the Hall current on an MHD natural convection flow with radiative heat transfer past a moving vertical plate with ramped temperature.

The objective of the present investigation is to study an unsteady natural convection transient flow of a viscous, incompressible and electrically conducting fluid with radiative heat transfer past an impulsively moving vertical plate embedded in a fluid saturated porous medium taking into account the effects of the Hall current and thermal diffusion when the temperature of the plate has a temporarily ramped profile.

2. Formulation of the problem and its solution

Consider an unsteady flow of a viscous, incompressible, electrically conducting and optically thick fluid past an infinite vertical plate embedded in a uniform porous medium. The coordinate system is chosen in such a way that the x' - axis is considered along the plate in upward direction and the y' - axis normal to the plane of the plate in the fluid. A uniform transverse magnetic field B_0 is applied in a direction which is parallel to the y' - axis. Initially, i.e., at time $t \le 0$, both the fluid and plate are at rest and have a uniform temperature T'_∞. At time $t > 0$, the plate starts moving in the x' - direction with uniform velocity U_0. The temperature of the plate is raised or lowered to $T'_\infty + (T'_w - T'_\infty)t'/t_0$ when $t' \le t_0$, and it is maintained at uniform temperature T'_w when $t' > t_0$ (t_0 being the characteristic time). The geometry of the problem is shown in Fig.1.

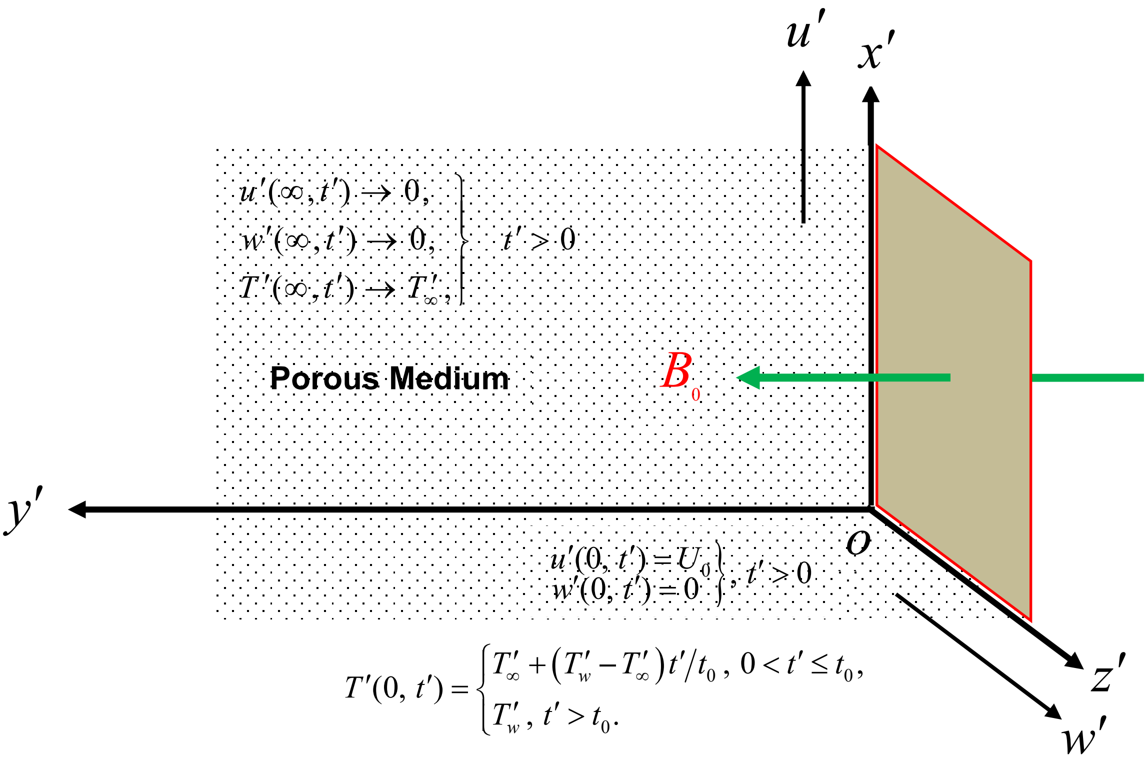

Fig.1. Geometry of the problem.

Since the plate is of infinite length in the x' and z' directions and is electrically non-conducting, all physical quantities except pressure, depend on y' and t' only. The induced magnetic field generated by the fluid motion is neglected in comparison to the applied one, i.e., the magnetic field $\boldsymbol{B} \equiv \left(0, B_0, 0\right)$. This assumption is valid because the magnetic Reynolds number is very small for liquid metals and partially ionized fluids (Cramer and Pai, 1973). Also no applied or polarized voltages exist so the effect of polarization of the fluid is negligible, i.e., the electric field $\boldsymbol{E} \equiv \left(0, 0, 0\right)$. This corresponds to the case where no energy is added or extracted from the fluid by electrical means (Cramer and Pai, 1973).

Keeping in view the assumptions made above, governing equations for the natural convection flow of a viscous, incompressible and electrically conducting fluid within a uniform porous medium with radiative heat transfer taking the Hall current into account, under the Boussinesq approximation, are given by

$$\frac{\partial u'}{\partial t'} = \upsilon \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho\left(1+m^2\right)}\left(u' + m w'\right) - \frac{\upsilon}{K_1'} u' + g\beta'\left(T' - T_\infty'\right), \tag{2.1}$$

$$\frac{\partial w'}{\partial t'} = \upsilon \frac{\partial^2 w'}{\partial y'^2} + \frac{\sigma B_0{}^2}{\rho\left(1+m^2\right)}\left(m u' - w'\right) - \frac{\upsilon}{K_1'} w', \tag{2.2}$$

$$\frac{\partial T'}{\partial t'} = \frac{k_1}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q_r'}{\partial y'}. \tag{2.3}$$

Initial and boundary conditions for the fluid-flow problem are

$$u' = w' = 0, \qquad T' = T_\infty' \qquad \text{for} \qquad y' \geq 0 \qquad \text{and} \qquad t' \leq 0, \tag{2.4a}$$

$$u' = U_0, \qquad w' = 0 \qquad \text{at} \qquad y' = 0 \qquad \text{for} \qquad t' > 0, \tag{2.4b}$$

$$T' = T_\infty' + \left(T_w' - T_\infty'\right) t'/t_0 \qquad \text{at} \qquad y' = 0 \qquad \text{for} \qquad 0 < t' \leq t_0, \tag{2.4c}$$

$$T' = T_w' \qquad \text{at} \qquad y' = 0 \quad \text{for} \qquad t' > t_0, \tag{2.4d}$$

$$u' \to 0, \quad w' \to 0, \qquad T' \to T_\infty' \qquad \text{as} \qquad y' \to \infty \qquad \text{for} \qquad t' > 0. \tag{2.4e}$$

For an optically thick fluid, in addition to emission there is also self-absorption and usually the absorption coefficient is wavelength dependent and large (Bestman, 1985) so we can adopt the Rosseland approximation for the radiative flux vector q_r' (Azzam, 2002). Thus q_r' is given by

$$q_r' = -\frac{4\sigma^*}{3k^*} \frac{\partial T'^4}{\partial y'}. \tag{2.5}$$

It is assumed that there is a small temperature difference between the fluid temperature T' and free stream temperature T_∞'. Equation (2.5) is linearized by expanding T'^4 in Taylor series about the free stream temperature T'_∞. Neglecting the second and higher order terms in $\left(T' - T'_\infty\right)$, T'^4 is expressed in the following form

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty . \tag{2.6}$$

Making use of Eqs (2.5) and (2.6) in Eq.(2.3), we obtain

$$\frac{\partial T'}{\partial t'} = \frac{k_1}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{1}{\rho c_p}\frac{16\sigma^* T'^3_\infty}{3k^*}\frac{\partial^2 T'}{\partial y'^2}. \tag{2.7}$$

Equations (2.1), (2.2) and (2.7), in a non-dimensional form, become

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \frac{M^2}{\left(1+m^2\right)}\left(u + mw\right) - \frac{u}{K_1} + \mathrm{G_r} T, \tag{2.8}$$

$$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial y^2} - \frac{M^2}{\left(1+m^2\right)}\left(mu - w\right) - \frac{w}{K_1}, \tag{2.9}$$

$$\frac{\partial T}{\partial t} = \frac{\left(1+N\right)}{\mathrm{P_r}}\frac{\partial^2 T}{\partial y^2} \tag{2.10}$$

where

$$y = y'/U_0 t_0, \qquad u = u'/U_0, \qquad w = w'/U_0, \qquad t = t'/t_0, \qquad T = \left(T' - T'_\infty\right)/\left(T'_w - T'_\infty\right),$$

$$M^2 = \sigma B_0^2 \upsilon / \rho U_0^2, \qquad K_1 = K'_1 U_0^2 / \upsilon^2, \qquad \mathrm{G_r} = g\beta'\upsilon\left(T'_w - T'_\infty\right)/U_0^3, \tag{2.11}$$

$$\mathrm{P_r} = \upsilon\rho c_p / k_1 \qquad \text{and} \qquad N = 16\sigma^* T'^3_\infty / 3k_1 k^* .$$

It is appropriate to mention here that the characteristic time t_0 is defined, according to the non-dimensional process mentioned above, as

$$t_0 = \upsilon / U_0^2 . \tag{2.12}$$

Initial and boundary conditions Eqs (2.4a) to (2.4e), in a non-dimensional form, become

$$u = w = 0, \qquad T = 0 \qquad \text{for} \qquad y \ge 0 \qquad \text{and} \qquad t \le 0, \tag{2.13a}$$

$$u = 1, \qquad w = 0 \qquad \text{at} \qquad y = 0 \qquad \text{for} \qquad t > 0, \tag{2.13b}$$

$$T = t \qquad \text{at} \qquad y = 0 \qquad \text{for} \qquad 0 < t \le 1, \tag{2.13c}$$

$$T = 1 \quad \text{at} \quad y = 0 \quad \text{for} \quad t > 1, \tag{2.13d}$$

$$u \to 0, \quad w \to 0, \quad T \to 0 \quad \text{as} \quad y \to \infty \quad \text{for} \quad t > 0. \tag{2.13e}$$

Combining Eqs (2.8) and (2.9), we obtain

$$\frac{\partial F}{\partial t} = \frac{\partial^2 F}{\partial y^2} - N^* F - \frac{1}{K_1} F + \mathrm{G_r} T \tag{2.14}$$

where $F = u + iw$ and $N^* = \dfrac{M^2 (1 - im)}{1 + m^2}$.

Initial and boundary conditions Eqs (2.13a) to (2.13e), in a compact form, are given by

$$F = 0, \quad T = 0 \quad \text{for} \quad y \geq 0 \quad \text{and} \quad t \leq 0, \tag{2.15a}$$

$$F = 1 \quad \text{at} \quad y = 0 \quad \text{for} \quad t > 0, \tag{2.15b}$$

$$T = t \quad \text{at} \quad y = 0 \quad \text{for} \quad 0 < t \leq 1, \tag{2.15c}$$

$$T = 1 \quad \text{at} \quad y = 0 \quad \text{for} \quad t > 1, \tag{2.15d}$$

$$F \to 0, \quad T \to 0 \quad \text{as} \quad y \to \infty \quad \text{for} \quad t > 0. \tag{2.15e}$$

Equations (2.10) and (2.14) with the use of the Laplace transform and initial conditions Eq.(2.15a) reduce to

$$\frac{d^2 \overline{T}}{dy^2} - sa\overline{T} = 0, \tag{2.16}$$

$$\frac{d^2 \overline{F}}{dy^2} - \left(s + N^* + \frac{1}{K_1} \right) \overline{F} + \mathrm{G_r} \overline{T} = 0 \tag{2.17}$$

where $a = \dfrac{\mathrm{P_r}}{(1 + N)}$, $\overline{F}(y,s) = \int\limits_0^\infty F(y,t)\, e^{-st} dt$ and $\overline{T}(y,s) = \int\limits_0^\infty T(y,t)\, e^{-st} dt$, ($s$ being Laplace transform parameter).

Boundary conditions Eqs (2.15b) to (2.15e), after taking the Laplace transform, become

$$\overline{F} = 1/s, \quad \overline{T} = \left(1 - e^{-s}\right)/s^2 \quad \text{at} \quad y = 0, \tag{2.18a}$$

$$\overline{F} \to 0, \quad \overline{T} \to 0 \quad \text{as} \quad y \to \infty. \tag{2.18b}$$

Equations (2.16) and (2.17), subject to the boundary conditions Eqs (2.18a) and (2.18b), are solved and the solution for $\overline{T}(y,s)$ and $\overline{F}(y,s)$ is given by

$$\bar{T}(y,s)=\frac{\left(1-e^{-s}\right)}{s^2}e^{-y\sqrt{as}}, \tag{2.19}$$

$$\bar{F}(y,s)=\frac{1}{s}e^{-y\sqrt{s+\lambda}}-\alpha\frac{\left(1-e^{-s}\right)}{s^2(s-\beta)}\left\{e^{-y\sqrt{s+\lambda}}-e^{-y\sqrt{as}}\right\} \tag{2.20}$$

where $\alpha=G_r/(a-1)$, $\lambda=N^*+(1/K_1)$ and $\beta=\lambda/(a-1)$.

An exact solution for the fluid temperature $T(y,t)$ and fluid velocity $F(y,t)$ is obtained by taking the inverse Laplace transform of Eqs (2.19) and (2.20) which is presented in the following form after simplification (Abramowitz and Stegun, 1972).

$$T(y,t)=G(y,t)-H(t-1)G(y,t-1), \tag{2.21}$$

$$\begin{aligned}F(y,t)=&\frac{1}{2}\left[e^{y\sqrt{\lambda}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{\lambda t}\right)+e^{-y\sqrt{\lambda}}\operatorname{erfc}\left(\frac{y}{2\sqrt{\lambda t}}-\sqrt{\lambda t}\right)\right]+\\&+\alpha\left[F^*(y,t)-H(t-1)F^*(y,t-1)\right]\end{aligned} \tag{2.22}$$

where

$$G(y,t)=\left(t+\frac{ay^2}{2}\right)\operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{a}{t}}\right)-\sqrt{\frac{at}{\pi}}ye^{-\frac{ay^2}{4t}},$$

$$\begin{aligned}F^*(y,t)=&\frac{1}{2}\Bigg[\frac{e^{\beta t}}{\beta^2}\Bigg\{e^{y\sqrt{(\lambda+\beta)}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{(\lambda+\beta)t}\right)+e^{-y\sqrt{(\lambda+\beta)}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{(\lambda+\beta)t}\right)+\\&-e^{y\sqrt{(\beta a)}}\operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{a}{t}}+\sqrt{\beta t}\right)-e^{-y\sqrt{(\beta a)}}\operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{a}{t}}-\sqrt{\beta t}\right)\Bigg\}+\\&-\frac{1}{\beta}\Bigg\{\left(t+\frac{1}{\beta}+\frac{y}{2\sqrt{\lambda}}\right)e^{y\sqrt{(\lambda)}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{\lambda t}\right)+\left(t+\frac{1}{\beta}-\frac{y}{2\sqrt{\lambda}}\right)-e^{-y\sqrt{(\lambda)}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{\lambda t}\right)+\\&-2\left(t+\frac{1}{\beta}+\frac{ay^2}{2}\right)\operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{a}{t}}\right)+2\sqrt{\frac{at}{\pi}}ye^{-\frac{ay^2}{4t}}\Bigg\}\Bigg].\end{aligned}$$

$H(t-1)$ and $\operatorname{erfc}(x)$ are, respectively, the unit step function and complementary error function.

3. Solution in case of isothermal plate

Solutions Eqs (2.21) and (2.22) present the analytical solution for the fluid temperature and fluid velocity for the flow of a viscous, incompressible, electrically conducting and optically thick fluid past an impulsively moving vertical plate with ramped temperature taking the Hall current, radiation and thermal diffusion into account. In order to know the influence of ramped temperature distribution within the plate on the fluid flow, it is appropriate to compare such a flow with the one past an impulsively moving vertical plate with uniform temperature. Keeping in view the assumptions made in the present study, a solution for the fluid temperature and fluid velocity for a flow past an impulsively moving vertical isothermal plate is obtained and is expressed in the following form

$$T(y,t) = \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{a}{t}}\right), \tag{3.1}$$

$$\begin{aligned} F(y,t) = {} & \frac{\left(1-d^*\right)}{2}\left[e^{y\sqrt{\lambda}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{\lambda t}\right)+e^{-y\sqrt{\lambda}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{\lambda t}\right)\right]+ \\ & +\frac{d^* e^{\beta t}}{2}\left[e^{y\sqrt{(\lambda+\beta)}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{(\lambda+\beta)t}\right)+e^{-y\sqrt{(\lambda+\beta)}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{(\lambda+\beta)t}\right)+\right. \\ & \left. -e^{y\sqrt{\beta a}}\operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{a}{t}}+\sqrt{\beta t}\right)-e^{-y\sqrt{\beta a}}\operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{a}{t}}-\sqrt{\beta t}\right)\right]+d^*\operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{a}{t}}\right) \end{aligned} \tag{3.2}$$

where $\qquad d^* = \alpha/\beta$.

4. Skin friction and Nusselt number

The expressions for the primary skin friction τ_x, secondary skin friction τ_z and Nusselt number Nu, which are measures of shear stress at the plate due to primary flow, shear stress at the plate due to secondary flow and rate of heat transfer at the plate respectively, are presented in the following form for ramped temperature and isothermal plates.

(i) For ramped temperature plate

$$\tau_x + i\tau_z = \sqrt{\lambda}\left(\operatorname{erfc}\left(\sqrt{\lambda t}\right)-1\right)-\frac{1}{\sqrt{\pi t}}e^{-\lambda t}+\alpha\left[F_1(y,t)-H(t-1)F_1(y,t-1)\right], \tag{4.1}$$

$$\mathrm{Nu} = 2\sqrt{\frac{a}{\pi}}\left\{\sqrt{t}-\sqrt{t-1}H(t-1)\right\} \tag{4.2}$$

where

$$\begin{aligned} F_1(y,t) = {} & \frac{e^{\beta t}}{\beta^2}\left\{\sqrt{(\lambda+\beta)}\left(\operatorname{erfc}\left(\sqrt{(\lambda+\beta)t}\right)-1\right)-\frac{1}{\sqrt{\pi t}}e^{-((\lambda+\beta)t)}-\sqrt{\beta a}\left(\operatorname{erfc}\left(\sqrt{\beta t}\right)-1\right)+\sqrt{\frac{a}{\pi t}}e^{-\beta t}\right\}+ \\ & -\frac{1}{\beta}\left[\left(t+\frac{1}{\beta}\right)\left\{\sqrt{\lambda}\left(\operatorname{erfc}\left(\sqrt{\lambda t}\right)-1\right)-\frac{1}{\sqrt{\pi t}}e^{-\lambda t}\right\}+\frac{1}{2\sqrt{\lambda}}\left(\operatorname{erfc}\left(\sqrt{\lambda t}\right)-1\right)+\sqrt{\frac{a}{\pi}}\left(2\sqrt{t}+\frac{1}{\beta\sqrt{t}}\right)\right]. \end{aligned}$$

(ii) For isothermal plate

$$\begin{aligned}\tau_x + i\tau_z = &\left(1-d^*\right)\left\{\sqrt{\lambda}\left(\mathrm{erfc}\left(\sqrt{\lambda t}\right)-1\right)-\frac{1}{\sqrt{\pi t}}e^{-\lambda t}\right\}+\\&+d^* e^{\beta t}\left[\sqrt{(\lambda+\beta)}\left\{\mathrm{erfc}\left(\sqrt{(\lambda+\beta)t}\right)-1\right\}-\frac{1}{\sqrt{\pi t}}e^{-(\lambda+\beta)t}+\right.\\&\left.-\sqrt{\beta a}\left\{\mathrm{erfc}\left(\sqrt{\beta t}\right)-1\right\}+\sqrt{\frac{a}{\pi t}}\right]-d^*\sqrt{\frac{a}{\pi t}},\end{aligned} \tag{4.3}$$

$$\mathrm{Nu}=\sqrt{\frac{a}{\pi t}}. \tag{4.4}$$

It is evident from the expressions Eqs (3.4) and (3.6) that, for a given time, the Nusselt number Nu is proportional to $\sqrt{a}\left(=\sqrt{\dfrac{\mathrm{P_r}}{N+1}}\right)$ in both the cases, i.e., the Nusselt number Nu increases on increasing the Prandtl number $\mathrm{P_r}$ while it decreases on increasing the radiation parameter *N*. Since $\mathrm{P_r}$ expresses the relative strength of viscosity to thermal diffusivity of the fluid, $\mathrm{P_r}$ decreases on increasing thermal diffusivity of the fluid. This implies that thermal diffusion and radiation tend to reduce the rate of heat transfer at both ramped temperature and isothermal plates. Also it is noticed from Eqs (3.4) and (3.6) that Nu increases for the ramped temperature plate whereas it decreases for the isothermal plate on increasing time *t*. This implies that, as time progresses, the rate of heat transfer at the ramped temperature plate is enhanced whereas it is reduced at the isothermal plate.

5. Results and discussions

In order to highlight the influence of various physical quantities, namely, the Hall current, thermal buoyancy force, permeability of medium, radiation and time on flow-field in the boundary layer region, the numerical values of fluid velocity, computed from the analytical solutions Eqs (2.22) and (3.2), are depicted graphically versus the boundary layer coordinate *y* in Figs 2 to 11 for various values of the Hall current parameter *m*, Grashof number $\mathrm{G_r}$, permeability parameter K_1, radiation parameter *N* and time *t* taking the magnetic parameter $M^2=15$ and Prandtl number $\mathrm{P_r}=0.71$. It is noticed from Figs 2 to 11 that, for both ramped temperature and isothermal plates, the primary velocity *u* and secondary velocity *w* attain a distinctive maximum value in the vicinity of the surface of the plate and then decrease properly on increasing the boundary layer coordinate *y* to approach the free stream value. Also, primary and secondary fluid velocities are faster in the case of the isothermal plate than that of ramped temperature plate. It is evident from Figs 2 to 11 that, for both ramped temperature and isothermal plates, the primary velocity *u* and secondary velocity *w* increase on increasing the Hall current parameter *m*, Grashof number $\mathrm{G_r}$, permeability parameter K_1, radiation parameter *N* and time *t*.

This implies that, for both ramped temperature and isothermal plates, the Hall current, thermal buoyancy force, permeability of the medium and radiation tend to accelerate the fluid flow in the primary and secondary flow directions in the boundary layer region. As time progresses, for both ramped temperature and isothermal plates, primary and secondary fluid velocities are getting accelerated in the boundary layer region.

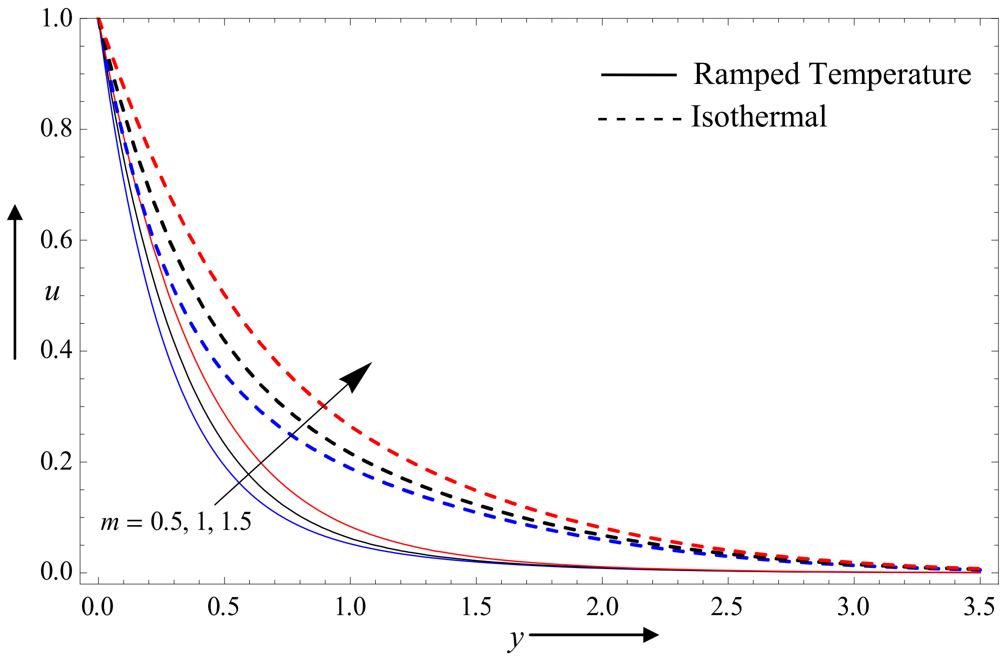

Fig.2. Primary velocity profiles when G_r=*6, K_1=0.5*, *N=1* and *t=0.4*.

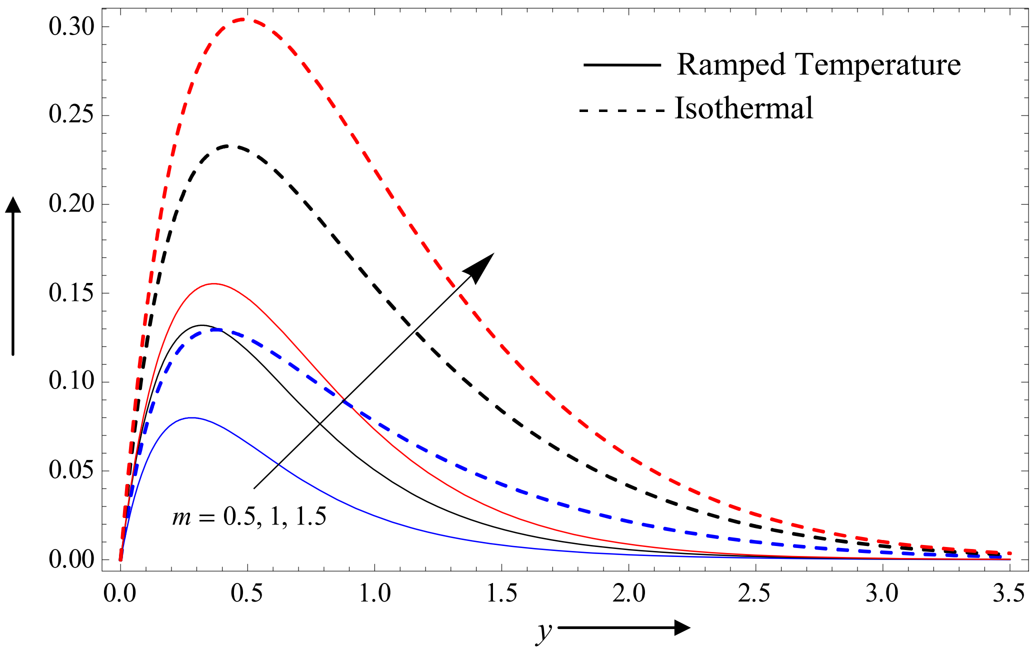

Fig.3. Secondary velocity profiles when G_r=*6*, *K_1=0.5*, *N=1* and *t=0.4*.

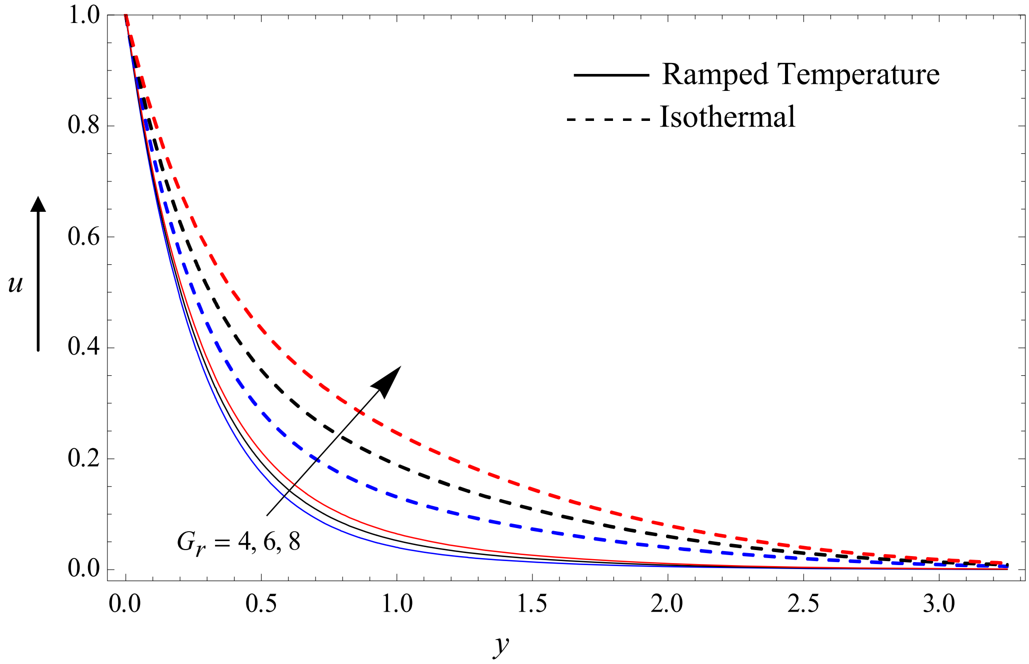

Fig.4. Primary velocity profiles when *m=0.5*, K_1*=0.5*, *N=1* and *t=0.4*.

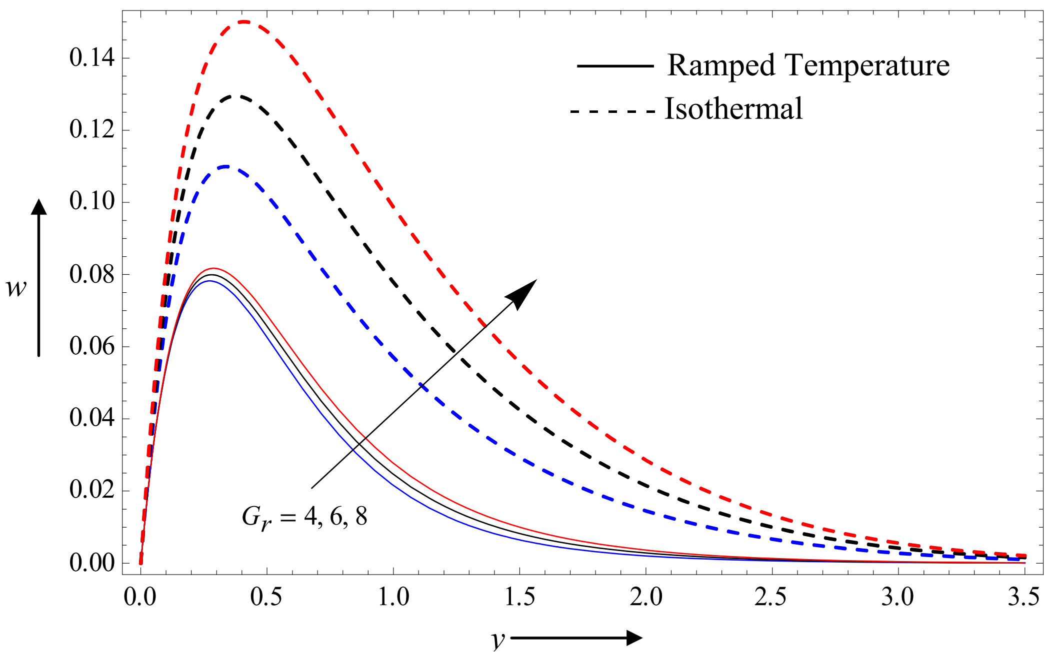

Fig.5. Secondary velocity profiles when *m=0.5*, K_1*=0.5*, *N=1* and *t=0.4*.

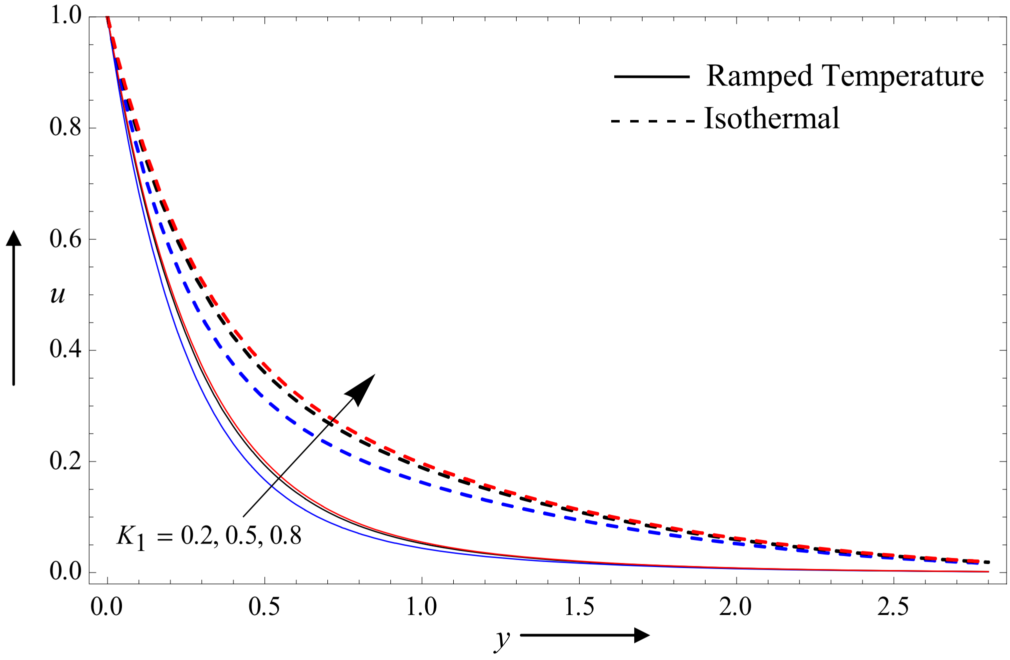

Fig.6. Primary velocity profiles when $m=0.5$, $G_r=6$, $N=1$ and $t=0.4$.

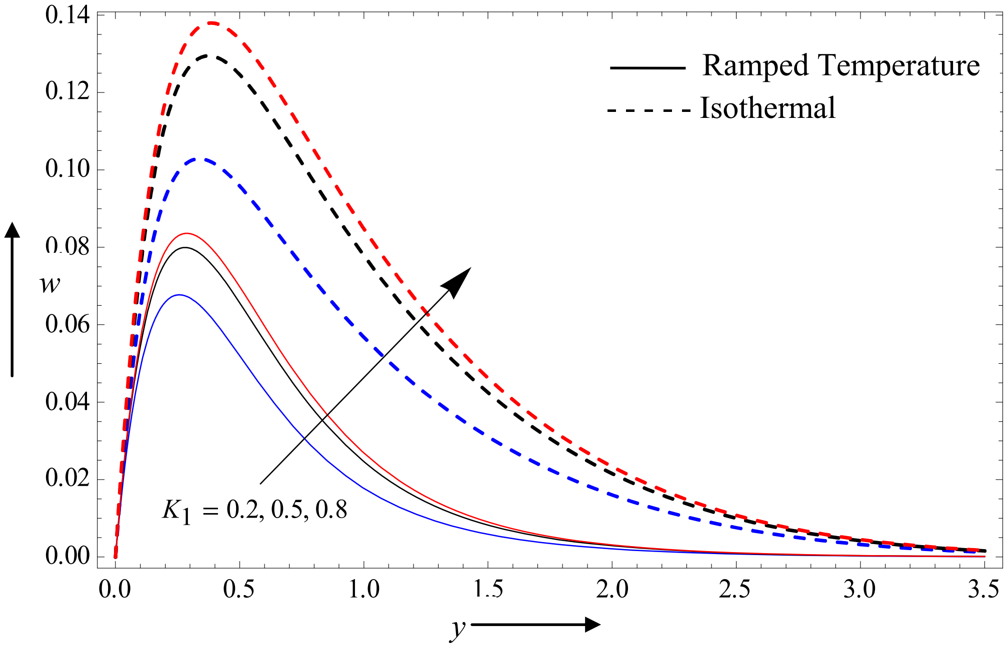

Fig.7. Secondary velocity profiles when $m=0.5$, $G_r=6$, $N=1$ and $t=0.4$.

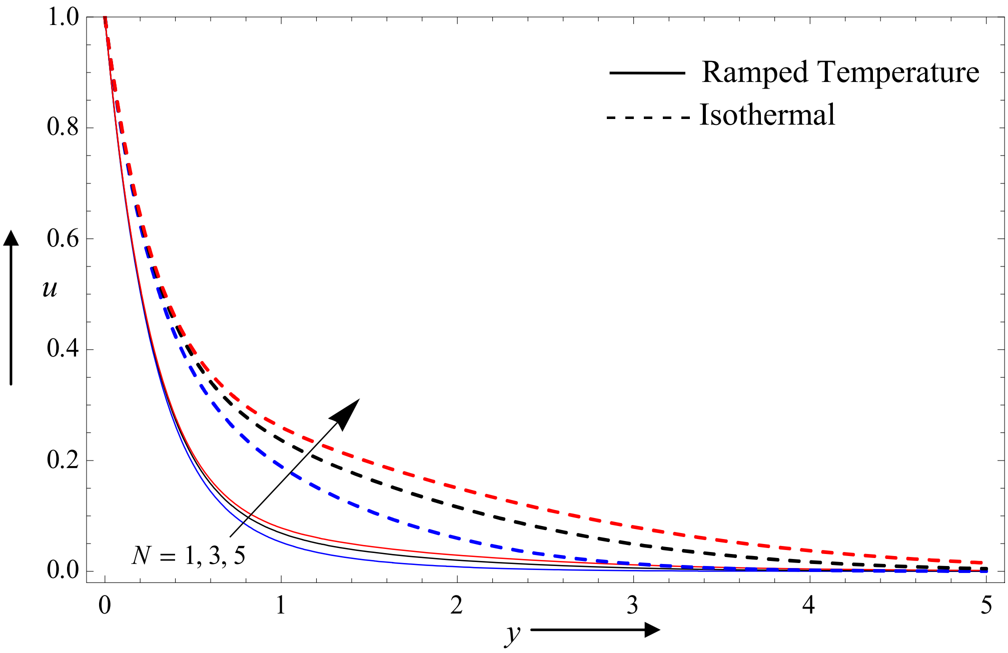

Fig.8. Primary velocity profiles when $m=0.5$, $K_1=0.5$, $G_r=6$ and $t=0.4$.

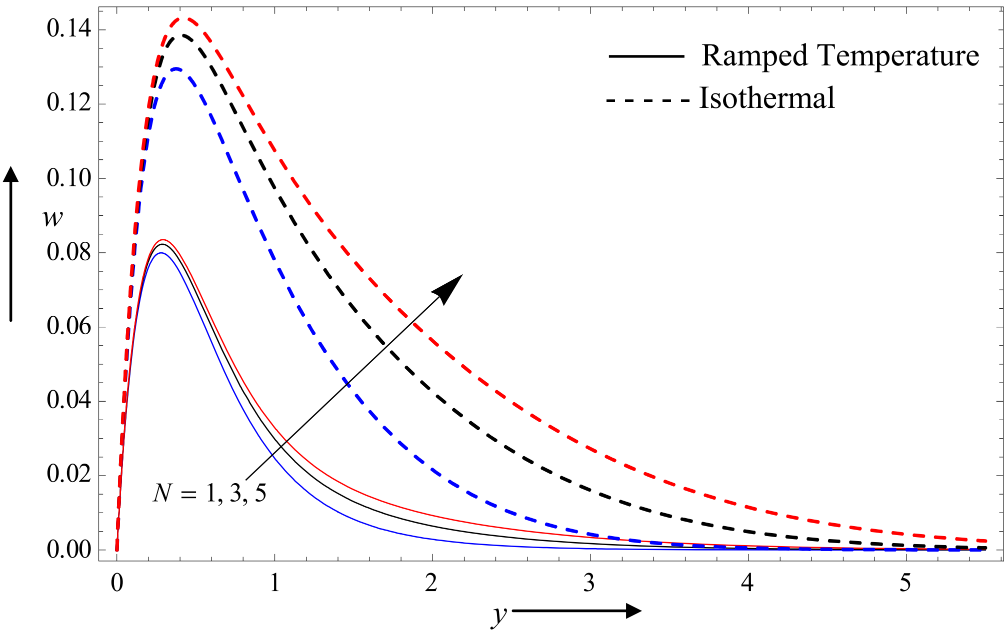

Fig.9. Secondary velocity profiles when $m=0.5$, $K_1=0.5$, $G_r=6$ and $t=0.4$.

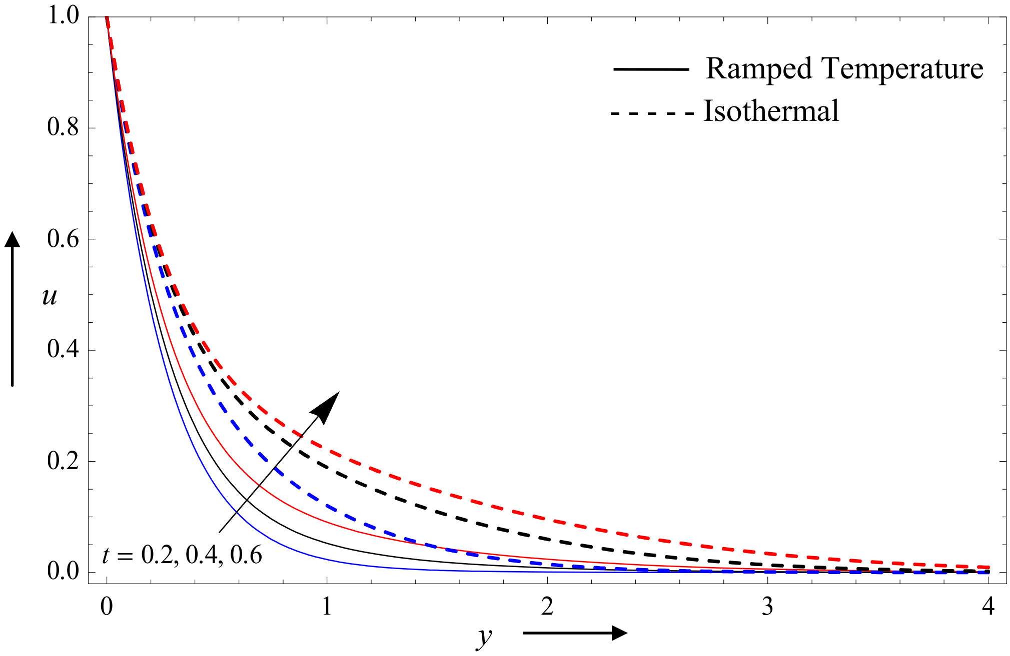

Fig.10. Primary velocity profiles when $m=0.5$, $K_1=0.5$, $G_r=6$ and $N=1$.

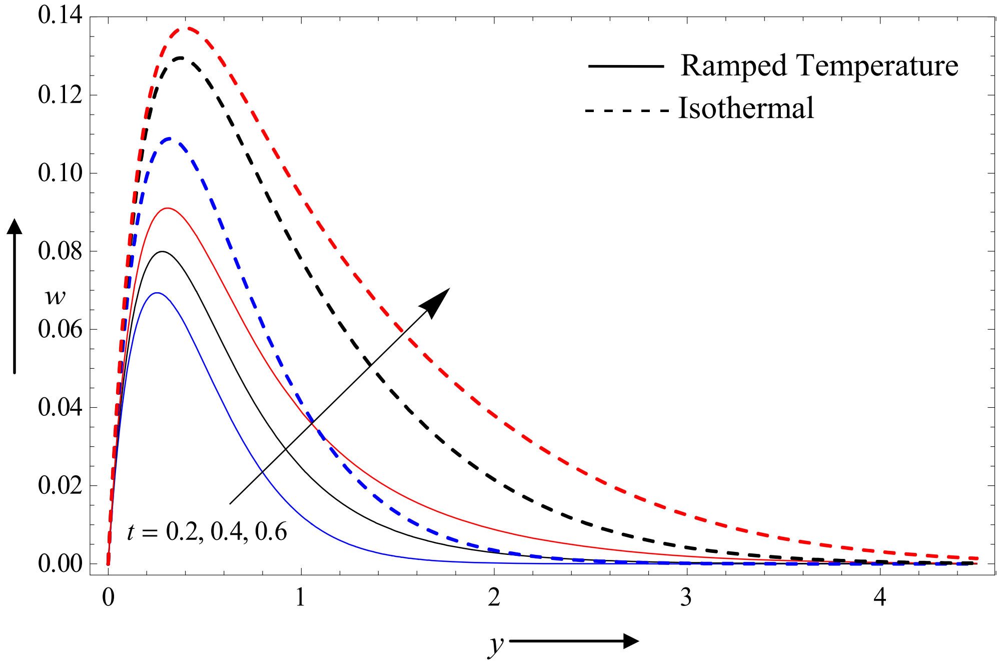

Fig.11. Secondary velocity profiles when $m=0.5$, $K_1=0.5$, $G_r=6$ and $N=1$.

In order to study the influence of radiation, thermal diffusion and time on the temperature field, numerical values of the fluid temperature T, computed from the analytical solutions Eqs (2.21) and (3.1), are displayed graphically versus the boundary layer coordinate y in Figs 12 to 14 for various values of N, P_r and t for both ramped temperature and isothermal plates. It is revealed from Figs 12 to 14 that, for both ramped temperature and isothermal plates, the fluid temperature T increases on increasing either the radiation parameter N or time t whereas it increases on decreasing the Prandtl number P_r. This implies that radiation and thermal diffusion tend to enhance the fluid temperature in the boundary layer region for both ramped temperature and isothermal plates. As time progresses, for both ramped temperature and isothermal plates, there is an enhancement in the fluid temperature in the boundary layer region. It is noticed from Figs 12 to 14 that the fluid temperature is maximum at the surface of the plate for both ramped temperature and isothermal plates and it decreases properly on increasing the boundary layer coordinate y to approach the free stream value. Also, the fluid temperature is lower for the ramped temperature plate than that for the isothermal plate.

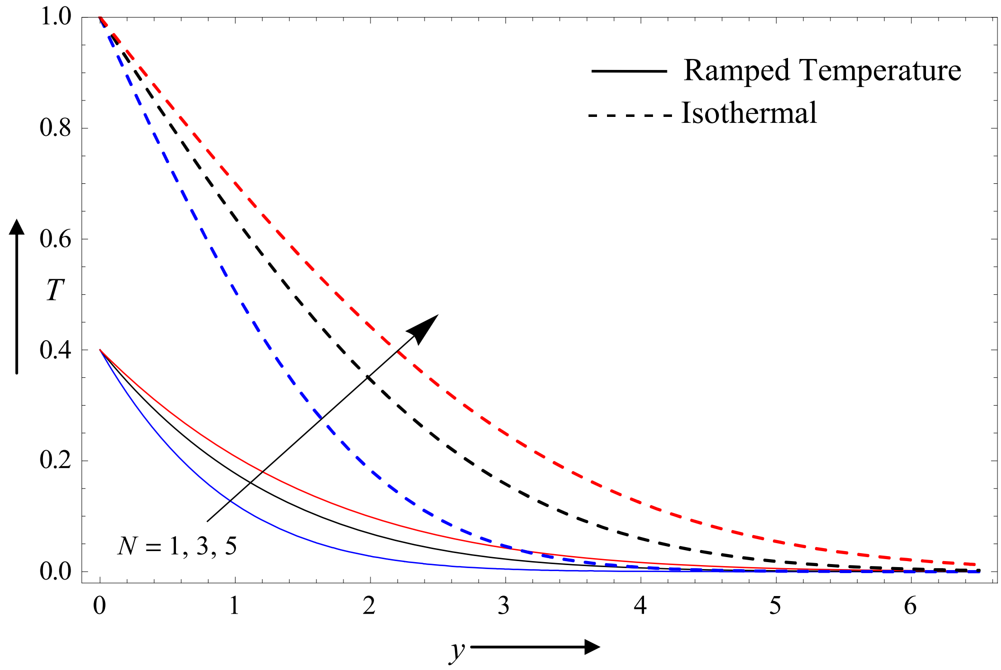

Fig.12. Temperature profiles when $t=0.4$ and $P_r=0.71$.

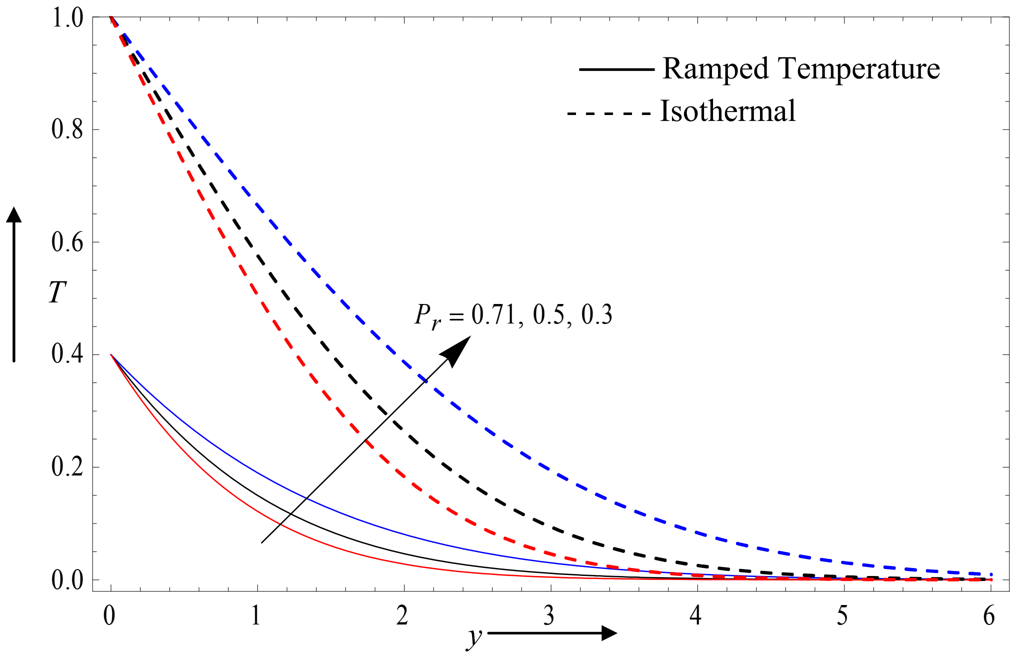

Fig.13. Temperature profiles when $t=0.4$ and $N=1$.

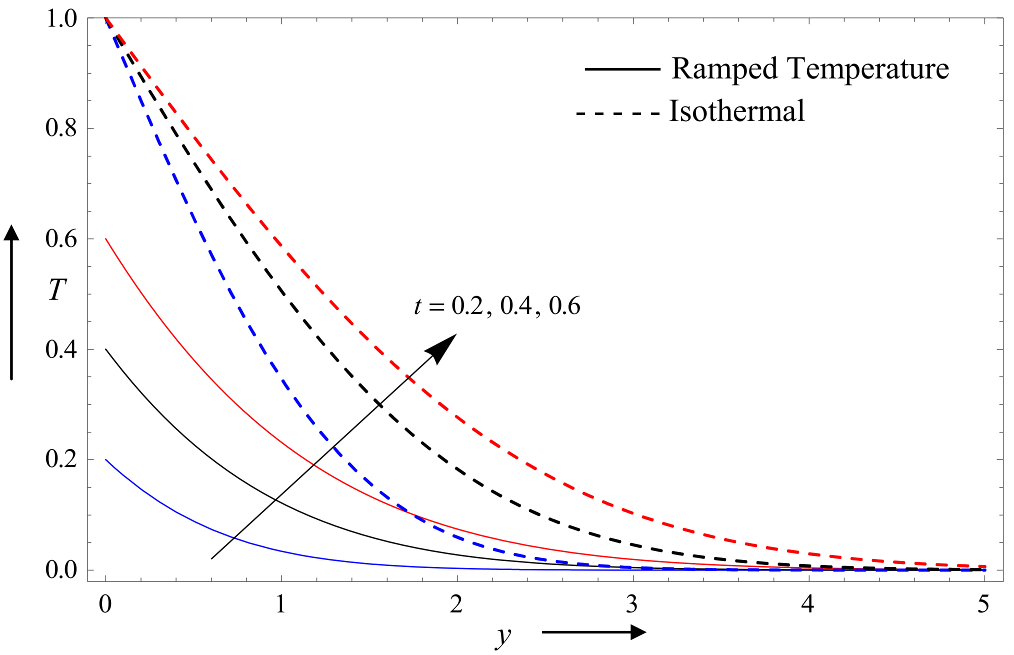

Fig.14. Temperature profiles when $N=1$ and $P_r=0.71$.

The numerical values of the primary skin friction τ_x and secondary skin friction τ_z for both ramped temperature and isothermal plates, computed from expressions Eqs (3.3) and (3.5), are presented in a tabular form in Tabs 1 to 6 for various values of m, G_r, K_1, N and t taking $M^2 = 15$ and $P_r = 0.71$. It is evident from Tabs 1 to 6 that, for both ramped temperature and isothermal plates, the primary skin friction τ_x decreases whereas the secondary skin friction τ_z increases on increasing m, G_r, K_1, N and t. This implies that, for both ramped temperature and isothermal plates, the Hall current, thermal buoyancy force, permeability of the medium and radiation tend to reduce the primary skin friction whereas these physical quantities have a reverse effect on the secondary skin friction. As time progresses, for both ramped temperature and isothermal plates, there is a reduction in the primary skin friction whereas there is an enhancement in the secondary skin friction.

Table 1. Skin friction at ramped temperature plate when $K_1=0.5$, $N=1$ and $t=0.4$.

$m \downarrow$ $G_r \rightarrow$	$-\tau_x$			τ_z		
	4	6	8	4	6	8
0.5	3.52773	3.38016	3.2326	0.822145	0.840905	0.859665
1	2.96664	2.80741	2.64819	1.2036	1.23458	1.26555
1.5	2.49560	2.32391	2.15222	1.29095	1.3273	1.36365

Table 2. Skin friction at isothermal plate when $K_1=0.5$, $N=1$ and $t=0.4$.

$m \downarrow$ $G_r \rightarrow$	$-\tau_x$			τ_z		
	4	6	8	4	6	8
0.5	3.08103	2.71011	2.3392	0.873748	0.918309	0.962871
1	2.47507	2.07006	1.66506	1.28524	1.35704	1.42884
1.5	1.94873	1.50359	1.05846	1.37326	1.45076	1.52826

Table 3. Skin friction at ramped temperature plate when $G_r=6$, $N=1$ and $t=0.4$.

$m\downarrow$ $K_1\rightarrow$	$-\tau_x$			τ_z		
	0.2	*0.5*	*0.8*	*0.2*	*0.5*	*0.8*
0.5	*3.76606*	*3.38016*	*3.27836*	*0.763887*	*0.840905*	*0.863658*
1	*3.2297*	*2.80741*	*2.69626*	*1.0949*	*1.23458*	*1.27702*
1.5	*2.7941*	*2.32391*	*2.1994*	*1.14304*	*1.3273*	*1.38548*

Table 4. Skin friction at isothermal plate when $G_r=6$, $N=1$ and $t=0.4$.

$m\downarrow$ $K_1\rightarrow$	$-\tau_x$			τ_z		
	0.2	*0.5*	*0.8*	*0.2*	*0.5*	*0.8*
0.5	*3.13249*	*2.71011*	*2.59802*	*0.831435*	*0.918309*	*0.943612*
1	*2.54491*	*2.07006*	*1.9422*	*1.19984*	*1.35704*	*1.40451*
1.5	*2.05595*	*1.50359*	*1.34866*	*1.25277*	*1.45076*	*1.51236*

Table 5. Skin friction at ramped temperature plate when $m=0.5$, $G_r=6$ and $K_1=0.5$.

$N\downarrow$ $t\rightarrow$	$-\tau_x$			τ_z		
	0.2	*0.4*	*0.6*	*0.2*	*0.4*	*0.6*
1	*3.63395*	*3.38016*	*3.11496*	*0.793177*	*0.840905*	*0.886539*
3	*3.61757*	*3.35039*	*3.07493*	*0.796584*	*0.849511*	*0.899521*
5	*3.60943*	*3.33604*	*3.05594*	*0.798411*	*0.853922*	*0.905993*

Table 6. Skin friction at isothermal plate when $m=0.5$, $G_r=6$ and $K_1=0.5$.

$N\downarrow$ $t\rightarrow$	$-\tau_x$			τ_z		
	0.2	*0.4*	*0.6*	*0.2*	*0.4*	*0.6*
1	*2.8951*	*2.71011*	*2.63569*	*0.823002*	*0.918309*	*0.954028*
3	*2.73568*	*2.5966*	*2.54322*	*0.887562*	*0.966504*	*0.993723*
5	*2.66206*	*2.54562*	*2.50195*	*0.916847*	*0.988774*	*1.01172*

6. Conclusions

A theoretical study of an unsteady MHD natural convection flow with radiative heat transfer past an impulsively moving vertical plate with ramped temperature in the presence of the Hall current and thermal diffusion is presented. Significant results are summarized below:

a. For both ramped temperature and isothermal plates:

The Hall current, thermal buoyancy force, permeability of the medium and radiation tend to accelerate the fluid flow in the primary and secondary flow directions in the boundary layer region. As time progresses, primary and secondary fluid velocities are getting accelerated in the boundary layer region.
Primary and secondary fluid velocities are faster in the case of the isothermal plate than that of the ramped temperature plate.

b. For both ramped temperature and isothermal plates:

Radiation and thermal diffusion tend to enhance the fluid temperature in the boundary layer region. As time progresses, there is an enhancement in the fluid temperature in the boundary layer region.

c. Fluid temperature is lower for the ramped temperature plate than for the isothermal plate.

d. For both ramped temperature and isothermal plates:

The Hall current, thermal buoyancy force, permeability of the medium and radiation tend to reduce the primary skin friction whereas these physical quantities have a reverse effect on the secondary skin friction. As time progresses, there is a reduction in the primary skin friction whereas there is an enhancement in the secondary skin friction.

Nomenclature

B_0 – uniform magnetic field
c_p – specific heat at constant pressure
$\mathrm{G_r}$ – Grashof number
g – acceleration due to gravity
K_1 – permeability parameter
K_1' – permeability of porous medium
k_1 – thermal conductivity
k^* – mean absorption coefficient
M^2 – magnetic parameter
$m = \omega_e \tau_e$ – Hall current parameter
N – radiation parameter
$\mathrm{P_r}$ – Prandtl number
q_r' – radiative flux vector
T' – fluid temperature
u', w' – fluid velocity in x' and z' - direction respectively
β' – coefficient of thermal expansion
ρ – fluid density
σ – electrical conductivity
σ^* – Stefan-Boltzmann constant
τ_e – electron collision time
υ – kinematic coefficient of viscosity
ω_e – cyclotron frequency

References

Abramowitz M. and Stegun I.A. (1972): *Handbook of Mathematical Functions*. – New York: Dover Publications, Inc.

Aldoss T.K., Al-Nimr M.A., Jarrah M.A. and Al-Shaer B.J. (1995): *Magnetohydrodynamic mixed convection from a vertical plate embedded in a porous medium*. – Heat Transfer, vol.28, pp.635-645.

Azzam G.E.A. (2002): *Radiation effects on the MHD mixed free forced convective flow past a semi-infinite moving vertical plate for high temperature differences*. – Phys. Scr., vol.66, pp.71-76.

Bestman A.R. (1985): *Free convection heat transfer to steady radiating non-Newtonian MHD flow past a vertical porous plate*. – Int. J. Methods Eng., vol.21, pp.899-908.

Cess R.D. (1966): *The interaction of thermal radiation with free convection heat transfer*. – Int. J. Heat Mass Transfer, vol.9, pp.1269-1277.

Chamkha A.J. (1997a): *Transient MHD free convection from a porous medium supported by a surface*. – Fluid/Particle Separation Journal, vol.10, pp.101-107.

Chamkha A.J. (1997b): *Hydromagnetic natural convection from an isothermal inclined surface adjacent to a thermally stratified porous medium*. – Int. J. Engng. Sci., vol.35, pp.975-986.

Chamkha A.J. (1997c): *Solar radiation assisted natural convection in a uniform porous medium supported by a vertical flat plate*. – ASME J. Heat Transfer, vol.119, pp.89-96.

Chamkha A.J. (2000): *Thermal radiation and buoyancy effects on hydromagnetic flow over an accelerating permeable surface with heat source or sink*. – Int. J. Engng. Sci., vol.38, pp.1699-1712.

Chamkha A.J., Takhar H.S. and Soundalgekar V.M. (2001): *Radiation effects on free convection flow past a semi-infinite vertical plate with mass transfer*. – The Chemical Engineering Journal, vol.84, pp.335-342.

Chang L.C., Yang K.T. and Lloyd J.R. (1983): *Radiation-natural convection interactions in two dimensional complex enclosures*. – ASME J. Heat Transfer, vol.105, pp.89-95.

Cheng P. and Minkowycz W.J. (1977): *Free convection about a vertical flat plate embedded in a porous medium with application to heat transfer from a dike*. – J. Geophys. Res., vol.82, pp.2040-2044.

Cookey C.I., Ogulu A. and Omubo-Pepple V.B. (2003): *Influence of viscous dissipation and radiation on unsteady MHD free-convection flow past an infinite heated vertical plate in a porous medium with time-dependent suction*. – Int. J. Heat Mass Transfer, vol.46, pp.2305-2311.

Cramer K.R. and Pai S.I. (1973): *Magnetofluid Dynamics for Engineers and Applied Physicists*. – New York: McGraw Hill Book Company.

Ede A.J. (1967): *Advances in Heat Transfer*. – New York: Academic Press, vol.4, pp.1-64.

Gebhart B. (1973): *Advances in Heat Transfer*. – NewYork: Academic Press, vol.9, pp.273-348.

Ghosh S.K. and Bég O.A. (2008): *Theoretical analysis of radiative effects on transient free convection heat transfer past a hot vertical surface in porous media*. – Nonlinear Anal. Modeling Control, vol.13, pp.419-432.

Hayday A.A., Bowlus D.A. and McGraw R.A. (1967): *Free convection from a vertical plate with step discontinuities in surface temperature*. – ASME J. Heat Transfer, vol.89, pp.244-250.

Hossain M.A. and Takhar H.S. (1996): *Radiation effects on mixed convection along a vertical plate with uniform surface temperature*. – Heat Mass Transfer, vol.31, pp.243-248.

Hsieh J.C., Chen T.S. and Armaly B.F. (1993): *Non-similarity solutions for mixed convection from vertical surfaces in porous media: variable surface temperature or heat flux*. – Int. J. Heat Mass Transfer, vol.36, pp.1485-1493.

Ibrahim F.S., Hassanien I.A. and Bakr A.A. (2004): *Unsteady magnetohydrodynamic micropolar fluid flow and heat transfer over a vertical porous medium in the presence of thermal and mass diffusion with constant heat source*. – Canad. J. Phys., vol.82, pp.775-790.

Jaluria Y. (1980): *Natural Convection Heat and Mass Transfer*. – Oxford: Pergamon Press.

Kao T.T. (1975): *Laminar free convective heat transfer response along a vertical flat plate with step jump in surface temperature*. – Lett. Heat Mass Transfer, vol.2, pp.419-428.

Kelleher M. (1971): *Free convection from a vertical plate with discontinuous wall temperature*. – ASME J. Heat Transfer, vol.93, pp.349-356.

Kim Y.J. (2000): *Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction*. – Int. J. Engng. Sci., vol.38, pp.833-845.

Lai F.C. and Kulacki F.A. (1991): *Non-Darcy mixed convection along a vertical wall in a saturated porous medium*. – J. Heat Transfer, vol.113, pp.252-254.

Lee S. and Yovanovich M.M. (1991): *Laminar natural convection from a vertical plate with a step change in wall temperature*. – ASME J. Heat Transfer, vol.113, pp.501-504.

Mahmoud Mostafa A.A. (2009): *Thermal radiation effect on unsteady MHD free convection flow past a vertical plate with temperature dependent viscosity*. – Canad. J. Chem. Eng., vol.87, pp.47-52.

Makinde O.D. (2009): *On MHD boundary layer flow and mass transfer past a vertical plate in a porous medium with constant heat flux*. – Int. J. Numer. Methods Heat Fluid Flow, vol.19, pp.546-554.

Makinde O.D. and Sibanda P. (2008): *Magnetohydrodynamic mixed convective flow and heat and mass transfer past a vertical plate in a porous medium with constant wall suction*. – J. Heat Transfer, vol.130, pp.112602-1-8.

Muthucumaraswamy R. and Ganesan P. (2003): *Radiation effects on flow past an impulsively started infinite vertical plate with variable temperature*. – Int. J. Appl. Mech. Eng., vol.8, pp.125-129.

Nakayama A. and Koyama H. (1987): *A general similarity transformation for combined free and forced convection flows within a fluid saturated porous medium*. – ASME J. Heat Transfer, vol.109, pp.1041-1045.

Nield D.A. and Bejan A. (2006): *Convection in Porous Media*. – USA: Springer, 3rd edn.

Ogulu A. and Makinde O.D. (2009): *Unsteady hydromagnetic free convection flow of a dissipative and radiating fluid past a vertical plate with constant heat flux*. – Chem. Eng. Commun., vol.196, pp.454-462.

Patra R.R., Das S., Jana R.N. and Ghosh S.K. (2012): *Transient approach to radiative heat transfer free convection flow with ramped wall temperature*. – J. Appl. Fluid Mech., vol.5, pp.9-13.

Pop I. and Ingham D.B. (2002): *Transport Phenomena in Porous Media-II*. – UK: Pergamon Press.

Raithby G.D. and Hollands K.G.T. (1985): *Natural convection* In: Handbook of Heat Transfer Fundamentals (W.M. Rohsenow *et al*. Ed.). – New York: McGraw Hill, 2nd edn.

Raptis A. (1986): *Flow through a porous medium in the presence of a magnetic field*. – Int. J. Energy Res., vol.10, pp.97-100.

Raptis A. and Kafousias N. (1982): *Heat transfer in flow through a porous medium bounded by an infinite vertical plate under the action of a magnetic field*. – Int. J. Energy Res., vol.6, pp.241-245.

Raptis A. and Massalas C.V. (1998): *Magnetohydrodynamic flow past a plate in the presence of radiation*. – Heat and Mass Transfer, vol.34, pp.107-109.

Sparrow E.M. and Cess R.D. (1970): *Radiation Heat Transfer*. – Brook/Cole, Belmont, California.

Vafai K. (2005): *Handbook of Porous Media*. – USA: Taylor and Francis Group, 2nd edn.

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